Fill in the following identities. **SCORE: _____ / 14 PTS**

[a] POWER REDUCING IDENTITY:

$\sin^2 x = \frac{1}{2}(1-\cos 2x)$

[b] HALF ANGLE IDENTITY:

$\cos\frac{1}{2}x = \pm\sqrt{\frac{1+\cos x}{2}}$

[c] PYTHAGOREAN IDENTITY:

$\tan^2 x = \sec^2 x - 1$

[d] NEGATIVE ANGLE IDENTITY:

$\sec(-x) = \sec x$

[e] DIFFERENCE OF ANGLES IDENTITY:

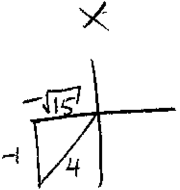
$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$

[f] SUM OF ANGLES IDENTITY:

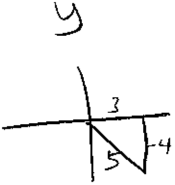
$\cos(x+y) = \cos x \cos y - \sin x \sin y$

[g] DOUBLE ANGLE IDENTITY: $\cos 2x = \cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ or $1 - 2\sin^2 x$

WRITE ALL 3 VERSIONS



x
$-\sqrt{15}$
-1
4



y
3
-4
5

$\tan \frac{1}{2} x$

$$= \frac{1 - \cos x}{\sin x}$$

$$= \frac{1 - \left(-\frac{\sqrt{15}}{4}\right)}{-\frac{1}{4}} \cdot \frac{-4}{-4}$$

$$= -4 - \sqrt{15}$$

$$\sin(\underbrace{\arctan(-\tfrac{4}{3})}_{y} - x)$$

$$= \sin y \cos x - \cos y \sin x$$

$$= -\frac{4}{5} \cdot -\frac{\sqrt{15}}{4} - \frac{3}{5} \cdot -\frac{1}{4}$$

$$= \frac{4\sqrt{15}+3}{20}$$

$\tan 2x$

$$= \frac{2\tan x}{1-\tan^2 x}$$

$$= \frac{2\left(\frac{1}{\sqrt{15}}\right)}{1-\left(\frac{1}{\sqrt{15}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{15}}}{\frac{14}{15}}$$

$$= \frac{2}{\sqrt{15}} \cdot \frac{15}{14}$$

$$= \frac{\sqrt{15}}{7}$$

Prove the identity $\cot^2 x + \sec^2 x = \sec^2 x \csc^2 x - 1$.

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$$= \sec^2 x(\cot^2 x + 1) - 1$$

$$= \sec^2 x \cot^2 x + \sec^2 x - 1$$

$$= \frac{1}{\cos^2 x} \frac{\cos^2 x}{\sin^2 x} - 1 + \sec^2 x$$

$$= \csc^2 x - 1 + \sec^2 x$$

$$= \cot^2 x + \sec^2 x$$

QED

Rewrite $\cos^4 x \sin^2 x$ using only the first powers of cosine (and constants and the 4 basic arithmetic operations). **SCORE: _____ / 14 PTS**

Simplify your final answer, which must **NOT** be in factored form, and must **NOT** involve any other trigonometric functions.

$$= \left[\tfrac{1}{2}(1+\cos 2x)\right]^2 \cdot \tfrac{1}{2}(1-\cos 2x)$$

$$= \tfrac{1}{8}(1+\cos 2x)(1+\cos 2x)(1-\cos 2x)$$

$$= \tfrac{1}{8}(1+\cos 2x)(1-\cos^2 2x)$$

$$= \tfrac{1}{8}(1+\cos 2x)(1-\tfrac{1}{2}(1+\cos 4x))$$

$$= \tfrac{1}{8}(1+\cos 2x)(\tfrac{1}{2}-\tfrac{1}{2}\cos 4x)$$

$$= \tfrac{1}{16}(1+\cos 2x)(1-\cos 4x)$$

$$= \tfrac{1}{16}+\tfrac{1}{16}\cos 2x-\tfrac{1}{16}\cos 4x-\tfrac{1}{16}\cos 2x\cos 4x$$

Solve the equation $2(3-\cos\frac{1}{4}x)=5-4\cos\frac{1}{4}x$.

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$$6-2\cos\tfrac{1}{4}x=5-4\cos\tfrac{1}{4}x$$

$$2\cos\tfrac{1}{4}x=-1$$

$$\cos\tfrac{1}{4}x=-\tfrac{1}{2}$$

$$\tfrac{1}{4}x=\frac{2\pi}{3}+2n\pi \text{ or } \frac{4\pi}{3}+2n\pi,\ n\in\mathbb{Z}$$

$$x=\frac{8\pi}{3}+8n\pi \text{ or } \frac{16\pi}{3}+8n\pi,\ n\in\mathbb{Z}$$

Solve the equation $4(1+\cos 2x) = 5(1-2\sin x)$ algebraically. **SCORE: _____ / 14 PTS**

Round your answers to 4 decimal places.

$4 + 4\cos 2x = 5 - 10\sin x$

$4 + 4(1 - 2\sin^2 x) = 5 - 10\sin x$

$8 - 8\sin^2 x = 5 - 10\sin x$

$0 = 8\sin^2 x - 10\sin x - 3$

$= (4\sin x + 1)(2\sin x - 3)$

$\sin x = -\frac{1}{4}$ OR $\frac{3}{2}$ (crossed out)

$x_{REF} = \sin^{-1}\frac{1}{4} \approx 0.2527$

$x \in Q_3$ OR Q_4

$x = \pi + 0.2527 + 2n\pi$

OR $2\pi - 0.2527 + 2n\pi$, $n \in \mathbb{Z}$

$= 3.3943 + 2n\pi$

OR $6.0305 + 2n\pi$, $n \in \mathbb{Z}$